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On Some Classes of 2-microhyperbolic systems

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§1. Introduction

Hereafter M denotes a real analytic manifold with a complexification X. We study a system of microdifferential equations $\mathfrak{M}$ defined in a neighborhood of $\rho_0 \in \mathring{T}^*_M X$. We assume that the characteristic variety of $\mathfrak{M}$ is written as

$\mathrm{Ch}(\mathfrak{M}) = \{\rho \in T^*X;\ p(\rho)=0\ \}$

by a homogeneous holomorphic function p defined in a neighborhood of ρ_0 satisfying the following conditions.

(1) p is real valued on $T^*_M X$.

(2) $\Sigma = \{\rho \in T^*_M X;\ p(\rho)=0,\ dp(\rho)=0\}$ is a regular involutory submanifold of $T^*_M X$ of codimension d through ρ_0.

(3) $\mathrm{Hess}(p)(\rho)$ has rank d with positivity 1.

The problem is to study the structure of $\mathcal{H}om_{\mathcal{E}_X}(\mathfrak{M}, \mathcal{C}_M)\big|_\Sigma$, the sheaf of microfunction solutions of $\mathfrak{M}$ on Σ.

§2 Canonical form

To express the canonical form, we take an open subset M_0 in $R_t^{n-d}\times R_x^d$ and a complex neighborhood X_0 of M_0 in $C_w^{n-d}\times C_z^d$. Then $(w,z;\theta dw+\xi dz)$ [resp. $(t,x;\sqrt{-1}(\tau dt+\xi dx))$] denotes a point of T^*X [resp. $T^*_M X$] with $\theta\in C^{n-d}$ and $\xi\in C^d$ [resp. $\tau\in R^{n-d}$ and $\xi\in R^d$].

By finding a suitable quantized contact transformation, the problem is reduced to study the system $\mathfrak{M}_0$ defined in a neighborhood of $\rho_0=(t=0,x=0;\sqrt{-1}dt_{n-d})$ whose characteristic variety is written as

$$Ch(\mathfrak{M})=\{(w,z;\theta,\xi)\in T^*X;\ \xi_1{}^2-\sum_{2\le i,j\le d} a_{ij}(w,z;\theta,\xi)\xi_i\xi_j=0\ \}.$$

Here a_{ij}'s are homogeneous holomorphic functions of order 0 defined in a neighborhood of ρ_0 and satisfy the condition

(4) $\qquad (a_{ij})_{2\le i,j\le d}$ is positive definite on $\Sigma_0=\{(t,x;\sqrt{-1}(\tau,\xi))\in T^*_{M_0}X_0;\ \xi=0\ \}.$

§3. Bisymplectic Structure due to Y. Laurent

To state the main theorem in an invariant form, we introduce the bisymplectic structure due to Y. Laurent[L].

Let Λ be a complexification of Σ in T^*X. By definition $\tilde{\Sigma}$ is the union of all bicharacteristic leaves of Λ issued from Σ. In case $\Sigma=\Sigma_0$, we may identify

(5) $$\tilde{\Sigma}_0 \simeq C^d{}_z\times\sqrt{-1}T^*R^{n-d}{}_{(t,\sqrt{-1}\tau dt)}.$$

Then we can take a coordinate of $T^*_{\Sigma_0}\tilde{\Sigma}_0$ as $(t,x;\sqrt{-1}\tau dt;\sqrt{-1}x^*dx)$ with $x^*\in R^d$.

We define a map

(6) $$p:\ T^*{}_{\Sigma}\tilde{\Sigma}\longrightarrow\Sigma\longrightarrow T^*{}_M X$$

and the canonical 1-form of $T^*{}_{\Sigma}\tilde{\Sigma}$ by $\omega_\Sigma=p^*\omega_M$. Here ω_M is a canonical 1-form of $T^*{}_M X$. We put $\Omega_\Sigma=d\omega_\Sigma$.

In case $\Sigma=\Sigma_0$, ω_Σ is written by coordinates as

$$\omega_\Sigma = \sum_j \tau_j dt_j.$$

We set

$$T_{rel}T^*_\Sigma\tilde{\Sigma} = \ker(TT^*_\Sigma\tilde{\Sigma} \longrightarrow T^*T^*_\Sigma\tilde{\Sigma}) \hookrightarrow TT^*_\Sigma\tilde{\Sigma}.$$

Here the morphism above in the definition of $T_{rel}T^*_\Sigma\tilde{\Sigma}$ is defined naturally by Ω_Σ. We dualize the exact sequence

$$0 \longrightarrow T_{rel}T^*_\Sigma\tilde{\Sigma} \longrightarrow TT^*_\Sigma\tilde{\Sigma}$$

and obtain

$$0 \longleftarrow T^*_{rel}T^*_\Sigma\tilde{\Sigma} \longleftarrow T^*T^*_\Sigma\tilde{\Sigma}.$$

We can take a section of $T^*_{rel}T^*_\Sigma\tilde{\Sigma}$ canonically, which is denoted by ω^r_Σ and called the relative canonical 1-form of $T^*_\Sigma\tilde{\Sigma}$. We also define the relative 2-form $\Omega^r_\Sigma = d\omega_\Sigma$.

In case $\Sigma = \Sigma_0$,

$$\omega^r_\Sigma = \sum_j x^*_j dx_j.$$

Associated with Ω^r_Σ we can define an isomorpfism

$$H^r_\Sigma:\ T^*_{rel}T^*_\Sigma\tilde{\Sigma} \xrightarrow{\ \sim\ } T_{rel}T^*_\Sigma\tilde{\Sigma}.$$

For a function f defined on an open subset of $T^*_\Sigma\tilde{\Sigma}$, we set

$$H^r_f = H^r_\Sigma(\ \overline{df}\ )$$

where $\overline{df}$ is the image of df by $T^*T^*_\Sigma\tilde{\Sigma} \longrightarrow T^*_{rel}T^*_\Sigma\tilde{\Sigma}$. H^r_f is called the relative Hamiltonian vector field of f.

In case $\Sigma=\Sigma_0$, it is written by coordinate as

$$H^r_f = \sum_j (\partial f/\partial x^*_j \cdot \partial/\partial x_j - \partial f/\partial x_j \cdot \partial/\partial x^*_j).$$

§4. 2-microfunctions

M. Kashiwara constructed the sheaf $\mathscr{C}^2_\Sigma$ of 2-microfunctions on $T^*_\Sigma\tilde{\Sigma}$ long time ago in Nice. We can study the properties of microfunctions defined on Σ precisely by $\mathscr{C}^2_\Sigma$. Explicitly, there exists the sheaf $\mathscr{C}_{\tilde{\Sigma}}$ of microfunctions along $\tilde{\Sigma}$ on $\tilde{\Sigma}$ and there exist the exact sequences

$$0 \longrightarrow \mathscr{C}_{\tilde{\Sigma}}\big|_\Sigma \longrightarrow \mathscr{B}^2_\Sigma \longrightarrow \pi_{\Sigma *}\left(\mathscr{C}^2_\Sigma\big|_{\dot{T}^*{}_\Sigma\tilde{\Sigma}} \right) \longrightarrow 0$$

and

$$0 \longrightarrow \mathscr{C}_M\big|_\Sigma \longrightarrow \mathscr{B}^2_\Sigma. \quad (\pi_\Sigma\colon T^*_\Sigma\tilde{\Sigma} \longrightarrow \Sigma.)$$

Here we put $\mathscr{B}^2_\Sigma = \mathscr{C}^2_\Sigma\big|_\Sigma$. Moreover we have the canonical spectral map

$$Sp^2_\Sigma\colon \ \pi_\Sigma^{-1}\mathscr{B}^2_\Sigma \longrightarrow \mathscr{C}^2_\Sigma.$$

We put for $u \in \mathscr{C}_M\big|_\Sigma$,

$$SS^2_\Sigma(u) = \mathrm{supp}(Sp^2_\Sigma(u)).$$

See Kashiwara-Laurent[K-L] for details about $\mathscr{C}^2_\Sigma$.

§5 Main Theorems

We set for a point $\rho \in \Sigma$ and $\tau \in T^*_\Sigma\tilde{\Sigma}\big|_\rho$

$$g = \langle \mathrm{Hess}(p)(\rho)H(\tau), H(\tau)\rangle$$

where $H\colon T^*_\Sigma\tilde{\Sigma} \longrightarrow T_\Sigma T^*_M X$ is an Hamiltonian isomorphism.

In case $\mathfrak{M} = \mathfrak{M}_0$,

$$g = x_1^{*2} - \sum_{2 \le i, j \le d} a_{ij}(t, x; \xi = 0, \tau) x_i^* x_j^*.$$

Here we give

Theorem 1. *Let u be a section of* $\mathscr{H}om_{\mathscr{E}_X}(\mathfrak{M}, \mathscr{C}_M)\big|_\Sigma$ *defined in a neighborhood of* ρ_0. *Then* $SS^2_\Sigma(u) \setminus \Sigma \subset \{ g = 0 \}$. *Moreover* $SS^2_\Sigma(u) \setminus \Sigma$ *is invariant under* H^r_g.

By Theorem 1 we can deduce a microlocal version of Holmgren's Theorem. We set

$$\gamma = \pi_{\Sigma}(\{\exp(sH_g^r)((\rho_0,\tau)); f(\rho_0,\tau)=0, s\geq 0 \}).$$

Here $\exp(sH_g^r)(q)$ denotes the flow of H_g^r issued from q. Then γ is a boundary of a cone in the bicharacteristic leave Γ of Σ through ρ_0. We take one of half-cones: γ_+. We give

Theorem 2. *Let u be a section of* $\mathcal{H}om_{\mathcal{E}_X}(\mathcal{M},\mathcal{C}_M)$ *defined in a neighborhood of* ρ_0. *Then*

$$\mathrm{supp}(u) \cap (\gamma_+\setminus\{\rho_0\}) = \phi$$

implies that $\rho_0 \notin \mathrm{supp}(u)$.

Here we remark that γ_+ does not contain the inside of the cone. Thus Theorem 2 generalizes the result of P. Laubin[Lb].

§6. Sketch of the proof.

6.1. As is mentioned in §2, it is enough to study the case $\mathfrak{M}=\mathfrak{M}_0$. Thus hereafter we put $\mathfrak{M}=\mathfrak{M}_0$, $X=X_0$, $M=M_0$, $\Sigma=\Sigma_0$ and

$$\Lambda=\Lambda_0=\{(w,z;\theta dw+\zeta dz)\in T^*X;\ \zeta=0 \}.$$

6.2. 2-microlocal canonical form ——*Single equations with a conditions on the lower terms*

6.2.1.

In case $\mathfrak{M}$ is reduced to a single equation : $Pu=0$ with $p=\sigma(P)$ satisfying the conditions (1),(2) and (3), we can transform the equation into a simple canonical form 2-microlocally if we assume $\mathfrak{M}$

has Regular Singularities along Λ in the sense of Kashiwara-Oshima [K-O].

6.2.2.

We embed Λ into $\Lambda\times\Lambda$ through the injection $T^*X — T^*_X(X\times X) \longrightarrow T^*(X\times X)$. By defintion, $\tilde{\Lambda}$ denotes the union of all bicharacteristic leaves of $\Lambda\times\Lambda$ passing through Λ. We take a coordinate of $T^*_\Lambda\tilde{\Lambda}$ as $(w,z;\theta dw;z^*dz)$ with $z^*\in\mathbf{C}^d$. On $T^*_\Lambda\tilde{\Lambda}$, Y. Laurent [L] defined the sheaf $\mathcal{E}^{2,\infty}_\Lambda$ of 2-microdifferential operators of infinite order.

Definition 3 (Y. Laurent[L]) Let Ω be an open subset of $T^*_\Lambda\tilde{\Lambda}$. Then $\sum_{i,j}P_{ij}(w,z;\theta;z^*)\in\mathcal{E}^{2,\infty}_\Lambda(\Omega)$ if and only if the following conditions (4) and (5) are satisfied.

(4) P_{ij} is holomorphic on Ω and homogeneous of order j with respect to (θ,z^*) and of order i with respect to z^*.

(5) For any compact subset K of Ω and for any positive number ε there exists a positive number $C_{\varepsilon,K}$ and for any compact subset K of Ω there exists a positive number C_K such that

$$\sup_K|P_{i,i+k}| \le \begin{cases} C_{\varepsilon,K}\,\varepsilon^{i+k}/i!\,k! & (i,k\ge 0) \\ C_{\varepsilon,K}^{-k}\,\varepsilon^{i}\,(-k)!/i! & (i\ge 0,k<0) \\ C_{\varepsilon,K}\,\varepsilon^{k}\,C_K^{-i}\,(-i)!/k! & (k\ge 0,i<0) \\ C_K^{-i-k}\,(-i)!\,(-k)! & (i,k<0). \end{cases}$$

We define the sheaf $\mathcal{E}^2_\Lambda$ of 2-microdifferential operators of finite order as follows.

Definition 4 For $P=\sum P_{ij}\in\mathcal{E}_{\Lambda}^{2,\infty}$, $P\in\mathcal{E}_{\Lambda}^{2}$ if and only if there exists j_0 such that

$$P_{ij}=0 \qquad (j>j_0)$$

and there exists $\lambda(j)$ for any $j\in\mathbf{Z}$ such that

$$P_{ij}=0 \qquad (i<\lambda(j)).$$

For any $P\in\mathcal{E}_{\Lambda}^{2}$, the principal symbol of P is defined by

$$\sigma_{\Lambda}(P) = P_{i_0 j_0}$$

where $j_0=\sup\{j;$ for some i $P_{ij}= 0\}$ and $i_0=\inf\{i;\ P_{ij_0}= 0\}$.

In the same way, we can construct the bisymplectic structure $(\Omega_{\Lambda},\Omega_{\Lambda}^{r})$. By coordinates, these are written as

$\Omega_{\Lambda}= \sum_j d\theta_j\wedge dw_j$ and $\Omega_{\Lambda}^{r} = \sum_j dz_j^{*}\wedge dz_j$.

If a map φ: $U \longrightarrow V$ between open subsets of $T^{*}{}_{\Lambda}\tilde{\Lambda}$ satisfies

$$\varphi^{*}(\Omega_{\Lambda}|_{V}) = \Omega_{\Lambda}|_{U} \quad ,$$

then we can induce an isomorphism

$$\varphi^{*}:\quad T_{rel}^{*}T^{*}{}_{\Lambda}\tilde{\Lambda}\ |_{V} \underset{V}{\times} U \longrightarrow T_{rel}^{*}T^{*}{}_{\Lambda}\tilde{\Lambda}|_{U}.$$

Moreover if

$$\varphi^{*}(\Omega_{\Lambda}^{r}|_{V}) = \Omega_{\Lambda}^{r}|_{U}$$

and φ preserves the bihomogeniety structure of $T^{*}{}_{\Lambda}\tilde{\Lambda}$:

$$(w,z;\theta;z^{*}) \longrightarrow (w,z;\lambda\theta;\lambda z^{*})$$

and

$$(w,z;\theta;z^{*}) \longrightarrow (w,z;\theta;\lambda z^{*}) \qquad (\lambda\in\mathbf{C}^{\times}),$$

then φ is called a homogeneous bicanonical transformation. Associated with φ, we can construct a ring isomorphism

$$\Phi:\quad \varphi^{-1}(\mathcal{E}_{\Lambda}^{2,\infty}|_{V}) \longrightarrow \mathcal{E}_{\Lambda}^{2,\infty}|_{U}.$$

See Y. Laurent[L] for details about 2-microdifferential operators.

6.2.3.

By finding a suitable quantized bicanonical transformation, we can transform the equation $Pu=0$ into

$$RP_0 u = 0$$

defined in a neighborhood of $\tau_0=(t=0,x=0;\sqrt{-1}dt_{n-d};\sqrt{-1}dx_d)$. Here

R is invertible at τ_0

and

$$\sigma_\Lambda(P_0) = z_1^*.$$

We remark that

$$S(P)=\{(j,i);\ P_{ij}\neq 0\ \} \subset \{\ i\geq j,\ j\leq 1\ \}.$$

Next we find an invertible 2-microdifferential operator of infinite order Q satisfying

$$QP_0 = D_1 Q.$$

Then we can easily prove Theorem 1. See [T_3] for details.

6.3. **2-microhyperbolicity —— general case**

6.3.1.

In general case, we prove Theorem 1 by employing the theory of microlocal analysis of sheaves due to Kashiwara-Schapira[K-S_2].

6.3.2.

Let X be an C^∞ manifold and let M be a closed submanifold of X in this section 6.3.2.

$D^+(X)$ denotes the derived category of bounded below complexes of sheaves of modules on X. For $\mathcal{F}\in Ob(D^+(X))$, $SS(\mathcal{F})$ denotes the microsupport of $\mathcal{F}$, which is a conic closed subset in T^*X.

For $\mathcal{F}\in Ob(D^+(X))$, $\mu_M(\mathcal{F})$ denotes Sato's microlocalization of $\mathcal{F}$

along M, which is an object of $D^+(T^*_M X)$.

For a closed subset of Z, $C_M(Z)$ denotes the normal cone of Z along M, which is a closed subset of $T_M X$.

We quote an important formula from Kashiwara-Schapira[K-S_2] as follows.

Theorem 5 *For* $\mathcal{F}\in \mathrm{Ob}(D^+(X))$, *we have*

$$SS(\ \mu_M(\mathcal{F})\ )\subset C_{T^*_M X}(\ SS(\mathcal{F})\ ).$$

Here we consider the right side as a subset of $T^*T^*_M X$ through

$$(-H):\ T_{T^*_M X} T^* X \xrightarrow{\ \sim\ } T^*T^*_M X.$$

(H is the Hamiltonian isomorphism.)

6.3.3.

We set $N=(\mathbf{R}^{n-d}_t\times\mathbf{C}^d)\cap X$ in X. Then we have

$$\mathscr{C}^2_\Sigma = \mu_\Sigma\mu_N(\ \mathcal{O}_X\ )[n].$$

Thus we can show by the theory of Kashiwara-Schapira[K-S_2] that

$$SS(R\mathcal{H}om_{\mathscr{E}_X}(\ \mathfrak{M},\ \mathscr{C}^2_\Sigma\ ))\subset C_{T^*_\Sigma\tilde{\Sigma}}(\ C_{T^*_N X}(\ Ch(\mathfrak{M})\ )\ ).$$

By estimating the right side, we can show

$$SS(R\mathcal{H}om_{\mathscr{E}_X}(\mathfrak{M},\ \mathscr{C}^2_\Sigma)\big|_{T^*_\Sigma\tilde{\Sigma}\setminus\Sigma})\subset\{(\rho,\tau)\in T^*(T^*_\Sigma\tilde{\Sigma}\setminus\Sigma);\ g(\rho)=0,\ \tau(H^r_g(\rho))=0\}$$

where $\rho\in T^*_\Sigma\tilde{\Sigma}\setminus\Sigma$ and $\tau\in T^*T^*_\Sigma\tilde{\Sigma}|_\rho$. Then we can easily prove Theorem 1 by Proposition 4.1.2 of [K-S_2].

§7. Some Remarks

7.0. We gather results for some classes of systems of microdifferential equations in §7.

7.1. *Case I*

Let M be a real analytic manifold with a complexification X. Let $\mathfrak{M}$ be a coherent $\mathcal{E}_X$ module defined in a neighborhood of $\rho_0 \in \mathring{T}^*_M X$ whose characteristic variety is written in a neighborhood of ρ_0 as

$$\mathrm{ch}(\mathfrak{M}) = \{\rho \in T^*X;\ p_1 = \cdots = p_{d-2} = 0,\ p_{d-1} \cdot p_d = 0\}$$

by homogeneous holomorphic functions $p_1, \cdot\cdot, p_{d-1}$ and p_d satisfying the following conditions.

(6) $p_1, \cdot\cdot, p_{d-1}$ and p_d are real valued on $T^*_M X$.

(7) $dp_1, \cdot\cdot, dp_{d-1}$ and dp_d and ω (canonical 1-form of T^*X) are linearly independent at ρ_0.

Let $\Lambda_1 = \{\rho \in \mathring{T}^*X;\ p_1 = \cdot\cdot = p_{d-1} = 0\}$, $\Lambda_2 = \{\rho \in \mathring{T}^*X;\ p_1 = \cdot\cdot = p_{d-2} = p_d = 0\}$ and $\Lambda = \Lambda_1 \cap \Lambda_2$. Then we assume

(8) Λ_1, Λ_2 and Λ is regular involutory submanifolds in $\mathring{T}^*X$ through ρ_0.

We set $\Sigma_i = T^*_M X \cap \Lambda_i$ $(i=1,2)$ and $\Sigma = \Sigma_1 \cap \Sigma_2$. Then the result is

Theorem 6.

Let u be a section of $\mathcal{H}om_{\mathcal{E}_X}(\mathfrak{M}, \mathcal{C}_M)$ defined in a neighborhood of ρ_0 and let Γ be the bicharacteristic leaf of Σ through ρ_0. Then there exist a family of bicharacteristic leaves of Σ_1 on Γ: $\{\gamma_s^{(1)}\}$ and that of Σ_2 on Γ: $\{\gamma_s^{(2)}\}$ such that

$$supp(u) = \cup_s \gamma_s^{(1)} \cup \cup_s \gamma_s^{(2)} \cup \{\textit{some of connected}$$

components of $[\ \Gamma\setminus(\bigcup_s \gamma^{(1)} \cup \bigcup_s \gamma_s^{(2)})\ ]\}$.

(sketch of the proof)

By finding a suitable quantized contact transformation, the problem is reduced to studying a coherent $\mathcal{E}_X$ module $\mathfrak{M}_0$ defined in a neighborhood of $\rho_0=(0,\sqrt{-1}dx_n)\in\sqrt{-1}\mathring{T}^*M_0$ whose characteristic variety is written as

(9) $$\mathrm{Ch}(\mathfrak{M})=\{(z,\xi dz)\in\mathring{T}^*X_0;\ \xi_1=\cdots=\xi_{d-2}=0,\ \xi_{d-1}\cdot\xi_d=0\}.$$

Here M_0 is an open subset of $R^n{}_x$ and X_0 is a complex neighborhood of M_0 in $\mathbf{C}^n{}_z$. Then $(z,\xi dz)$ [resp. $(x,\sqrt{-1}\xi dx)$] denotes a point of T^*X_0 [resp. $T^*{}_{M_0}X_0$] with $\xi\in\mathbf{C}^n$ [resp. $\xi\in R^n$]. We set

$$\Sigma_0=\{(x,\sqrt{-1}\xi dx)\in\sqrt{-1}T^*M_0;\ \xi_1=\cdots=\xi_d=0\ \}$$

and take a coordinate of $T^*{}_{\Sigma_0}\tilde{\Sigma}_0$ as $(x,\sqrt{-1}\xi'';\sqrt{-1}x'^*)$ with $\xi''=(\xi_{d+1},\cdots,\xi_n)$ and $x'^*=(x_1^*,\cdots,x_d^*)$. Then for a section u of $\mathcal{H}om_{\mathcal{E}_{X_0}}(\mathfrak{M}_0,\mathcal{C}_M)$ defined in a neighborhood of ρ_0, we have

(10) $$SS^2_{\Sigma_0}(u)\setminus\Sigma_0\subset\{\ x_1^*=\cdots=x_{d-1}^*=0\}\ \cup\ \{\ x_1^*=\cdots=x_{d-2}^*=x_d^*=0\ \}.$$

We set

$$\Gamma_1=\{(x,\sqrt{-1}\xi'';\sqrt{-1}x'^*)\in T^*{}_{\Sigma_0}\tilde{\Sigma}_0\setminus\Sigma_0;\ x_1^*=\cdots=x_{d-1}^*=0\ \}$$

and

$$\Gamma_2=\{(x,\sqrt{-1}\xi'';\sqrt{-1}x'^*)\in T^*{}_{\Sigma_0}\tilde{\Sigma}_0\setminus\Sigma_0;\ x_1^*=\cdots=x_{d-2}^*=x_d^*=0\ \}.$$

Then $SS^2_{\Sigma_0}(u)\big|_{\Gamma_1}$ [resp. $SS^2_{\Sigma_0}(u)\big|_{\Gamma_2}$] is invariant under the integrable syatem $(\partial/\partial x_1,\cdots,\partial/\partial x_{d-1})$ [resp. $(\partial/\partial x_1,\cdots,\partial/\partial x_{d-2},\partial/\partial x_d)$]. This fact is shown in the same way as in §6.3.

(q.e.d.)

7.2. *(Case II)*

Let M be a real analytic manifold with a complexification X. Let $\mathfrak{M}$ be a coherent $\mathcal{E}_X$ module defined in a neighborhood of $\rho_0 \in \mathring{T}^*{}_M X$ whose characteristic variety is written in a neighborhood of ρ_0 as

(11) $\quad Ch(\mathfrak{M}) = \{\rho \in T^*X;\ p=0\}$

by a homogeneous holomorphic function p satisfying the following conditions.

(12) $\quad$ p is real valued on $T^*{}_M X$.

(13) $\quad \Sigma = \{\rho \in \mathring{T}^*{}_M X;\ p(\rho)=0,\ dp(\rho)=0\}$ is a regular involutory submanfold of codimension 2 in $T^*{}_M X$ through ρ_0.

(14) $\quad$ Hess(p)(ρ) has rank 1 if $\rho \in \Sigma$.

We set for a point $\rho \in \Sigma$ and $\tau \in T^*{}_\Sigma \tilde{\Sigma}\big|_\rho$

(15) $\quad g = \langle \mathrm{Hess}(p)(\rho) H(\tau), H(\tau) \rangle$

where H: $T^*{}_\Sigma \tilde{\Sigma} \xrightarrow{\sim} T_\Sigma T^*{}_M X$ is Hamiltonian isomorphism. Then we have

Propostion 7.

The function g is divided into

$$g = g_1 \cdot g_2{}^2$$

with $g_1 \not\equiv 0$ on $T^*{}_\Sigma \tilde{\Sigma} \setminus \Sigma$.

By the decomposition above, we have

Theorem 8.

Let u be a section of $\mathcal{H}om_{\mathcal{E}_X}(\mathfrak{M}, \mathcal{C}_M)$ *defined in a neighborhood of* ρ_0. *Then*

$$SS^2_\Sigma(u) \setminus \Sigma \subset \{g_2 = 0\}.$$

Moreover $SS^2_\Sigma(u)\setminus\Sigma$ *is invariant under* $H^r_{g_2}$.

(sketch of the proof)

By finding a suitable quantized contact transformation, the problem is reduced to studying the system $\mathfrak{M}_0$ defined in a neighborhood of $\rho_0=(0,\sqrt{-1}dx_n)\in\sqrt{-1}T^*M_0$ whose characteristic variety is written as

(16) $$Ch(\mathfrak{M}_0)=\{(z,\zeta dz)\in T^*X_0;\ \zeta_1^2 - a(z,\zeta')\zeta_2^3 = 0\}.$$

Here M_0 is an open subset of $R^n{}_x$ and X_0 is a complex neighborhood of M_0 in X_0. Then $(z,\zeta dz)$ [resp. $(x,\sqrt{-1}\xi dx)$] denotes a point of T^*X_0 [resp. $T^*_{M_0}X_0 \simeq \sqrt{-1}T^*M_0$]. Moreover $a(z,\zeta')$ is a homogeneous holomorphic function of order (-1) with $\zeta'=(\zeta_2,\cdots,\zeta_n)$.

In this case,

$$\Sigma=\{(x,\sqrt{-1}\xi dx)\in\sqrt{-1}\mathring{T}^*M_0;\ \xi_1=\xi_2=0\ \}.$$

When we take a coordinate of $T^*_\Sigma\tilde{\Sigma}$ as $(x,\sqrt{-1}\xi'';\sqrt{-1}(x_1^*,x_2^*))$ with $\xi''=(\xi_3,\cdots,\xi_n)$, we can take g_2 as x_1^*. Then in the same way as in §6.3 we have

$$SS(R\mathcal{H}om_{\mathcal{E}_X}(\mathfrak{M},\mathcal{C}^2_\Sigma)\Big|_{T^*_\Sigma\tilde{\Sigma}\setminus\Sigma})$$
$$\subset\ \{(\rho,\tau)\in T^*(T^*_\Sigma\tilde{\Sigma}\setminus\Sigma);\ x_1^*(\rho)=0,\ \text{and}\ \tau(H^r(x_1^*))\ \}.$$

Here $\rho\in T^*_\Sigma\tilde{\Sigma}\setminus\Sigma$ and $\tau\in T^*(T^*_\Sigma\tilde{\Sigma}\setminus\Sigma)$. In the same way as in §6.3, $SS^2_\Sigma(u)$ is invariant under $\partial/\partial x_1$ for any section of $\mathcal{H}om_{\mathcal{E}_X}(\mathfrak{M},\mathcal{C}_M)$.

(q.e.d.)

We can easily show that Σ is foliated by the projection of the integral curves of $H^r_{g_2}$ in $\{g_2=0\}$. More precisely, for $\rho\in\Sigma$

$$\gamma(\rho)=\pi_{\Sigma}(\{\exp(H^{r}_{g_2})(\rho,\tau);\ g_2(\rho,\tau)=0\ ,\ s\in R\ \})$$

$$(\pi_{\Sigma}:\ T^{*}_{\Sigma}\tilde{\Sigma}\setminus\Sigma \longrightarrow \Sigma)$$

is a smooth curve in the bicharacteristic leaf Γ of Σ. Here we give

Theorem 9.

For any section u of $\mathcal{H}om_{\mathcal{E}_X}(\mathfrak{M},\mathcal{C}_M)|_{\Sigma}$ *defined in a neighborhood of* ρ_0, *supp(u)*$\cap\Sigma$ *propagates along the family of integral curves* $\{\gamma(\rho);\rho\in\Sigma\}$.

Remark 10. Theorem 9 itself can be proved by the microlocal version of Holmgren's theorem due to J.M. Bony[B].

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On some classes of 2-microhyperbolic systems

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Mを実解析的多様体, Xをその複素近傍とする。ここでは、$\mathring{T}^*_M X$の点ρ_0の近傍で定義されたsystem of microdifferential equations $\mathcal{M}$で、その特性多様体がρ_0の近傍で斉次な正則函数pを用いて

$$ch(\mathcal{M}) = \{\rho \in \mathring{T}^* X \ ; \ p(\rho) = 0\}$$

と書けるものを考える。但し、pは以下の条件を満たすものとする。

(1) pは$T^*_M X$上実数値

(2) $\Sigma = \{\rho \in \mathring{T}^*_M X ; \ p(\rho) = 0, \ dp(\rho) = 0\}$は$T^*_M X$中余次元$d$の正則包合的部分多様体で$\rho_0$を通るものである。

(3) $\mathrm{Hess}(p)(\rho)$は$\rho \in \Sigma$の時 rank dで positivity 1を持つ。

問題は、$\mathcal{M}$のmicro函数解のΣ上での構造である。Σに沿って、2超局所化して精密にこれを調べる。